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Abstract

The objective of this work is the development of efficient techniques for preliminary optimization of the cost associated with transfer trajectories to libration point orbits in the Sun-Earth-Moon four body problem; such transfers may also include lunar gravity assists. Initially, dynamical systems theory is used to determine invariant manifolds associated with the desired libration point orbit. These manifolds are employed to produce an initial approximation to the transfer trajectory. Specific trajectory requirements such as, transfer injection constraints, inclusion of phasing loops, and targeting of a specified state on the manifold are then incorporated into the design of the transfer trajectory. A two level differential corrections process is used to produce a fully continuous trajectory that satisfies the design constraints, and includes appropriate lunar and solar gravitational models. Based on this methodology, and using the manifold structure from dynamical systems theory, a technique is presented to optimize the cost associated with insertion onto a specified libration point orbit.

INTRODUCTION

Based on recent successes, a number of missions have lately been proposed that aim to take advantage of the growing scientific interest in the region of space near the libration points in the Sun-Earth system. To support missions that include increasingly complex trajectories and incorporate libration point orbits, more efficient techniques and new design philosophies must be considered. Typically, the first challenge in the design process to support a libration point mission is the numerical determination of the periodic or quasi-periodic orbit (i.e., the halo or Lissajous trajectory) that serves as the operational base to

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meet the scientific objectives of the mission. Design capabilities for these types of trajectories have significantly improved in the last few years, and baseline concepts derived from solutions in a three-body regime have been successfully exploited, since the first libration point mission in the late 1970's.[1–9]

Determination of a nominal halo or Lissajous trajectory is only one part of the design process, however. Transfer trajectories to and from this region of space must also be considered. For any trajectory problem, the ultimate goal is an analytical solution (or, at least, an analytical approximation), however, there has yet to be any significant progress in generating a closed form solution for transfers to and from the vicinity of the libration points. Frequently, the nominal halo or Lissajous trajectory is computed in conjunction with the transfer path, using straightforward propagation from Earth launch conditions. A more optimal approach involves the initial identification and design of a particular halo or Lissajous trajectory to closely match the specifications of the mission; then, the best transfer path from Earth, or the most useful trajectory arc to or from another point in this region is determined. (This latter approach is, in fact, crucial to the success of the trajectory design for the upcoming Genesis sample return mission.[10] Since the return to Earth places conditions and constraints on the nominal Lissajous trajectory, it is absolutely necessary that the launch leg is computed independently of the libration point orbit and return trajectory.) The recent introduction of certain aspects of Dynamical Systems Theory (DST) as a means of dynamical analysis and design in the three-body problem is motivated, in part, by the absence of any analytical tools for the computation of transfer trajectories, and by the requirement to determine Earth-launch-to-halo-orbit transfer paths in the context of the three-body problem.[6,10–13] The previous trial-and-error numerical search methods have clearly been successful in the past to compute these launch segments, but more efficient procedures that exploit knowledge of the dynamics are desirable. Application of DST in the circular restricted three-body problem yields a relatively fast method for generating a number of different types of trajectories to and from halo orbits, e.g., transfers between Earth and halo orbits, as well as, transfers between halo orbits in the vicinity of different libration points. An additional benefit of DST is a better understanding of the geometry of the phase space; this knowledge allows mission designers to obtain valuable insight into the behavior of solutions in this region of space.

Given an appropriate libration point trajectory, the additional information that a halo orbit in the circular restricted problem possesses one-dimensional stable manifolds, allows an initial estimate of a path from Earth to the periodic halo to be determined. This initial approximation is ultimately used to generate an Earth-to-halo transfer that closely reflects a manifold path, but in the more complex model that represents the "real" solar system. It has been shown previously that there exist stable manifolds that pass close to the Earth prior to an asymptotic approach to the halo or Lissajous trajectory in the vicinity of a libration point.[6,10–13] The point along such a stable manifold that represents the closest approach to the Earth serves as an ideal location for the insertion maneuver from Earth launch. Unfortunately, the state on the manifold path at closest approach rarely meets the constraints associated with actual launch conditions. Given the stable manifold (or other appropriate approximation), the goal then becomes the determination of the optimal location and magnitude of an insertion maneuver to successfully place the spacecraft on a path to complete the transfer into the libration point orbit. A methodology to successfully generate a suitable transfer that does, in fact, accomplish the task of inserting the spacecraft into the libration point orbit is now available.[10,14] The goal, then, of the current work is the extension of the previous procedure to produce a transfer path that is more optimal in terms of insertion cost. A secondary, perhaps equally important goal, is the additional understanding of the dynamics in this region to support future error analysis studies. In the following discussion, the procedure to produce the baseline transfer is presented first. Then, the extensions to the process to obtain more cost effective results are discussed and implemented in the algorithm. A number of examples are presented to demonstrate the application of this technique and its impact on the insertion maneuver.

BACKGROUND

Stable Manifolds

The procedure to determine a suitable transfer trajectory begins by examining the underlying structure of a given libration point orbit (LPO) using dynamical systems theory. From DST,[6,10,13] it is known that the stable and unstable manifolds associated with periodic and quasi-periodic solutions (such as libration point orbits) define subspaces in the six-dimensional phase space (position plus velocity). The concept of a manifold is simply a collection of orbits that start on a surface (i.e., in a subspace) and stay on that surface throughout their evolution. The computation of the stable and unstable manifolds corresponding to a periodic halo orbit in the circular problem is associated with properties of its monodromy matrix. Since the monodromy matrix possesses one stable and one unstable eigenvalue, the corresponding manifolds are one-dimensional in the phase space. The one-dimensional stable and unstable manifolds, then, are approximated by the corresponding eigenspaces, and can be computed at various points along the periodic orbit. This set of one-dimensional manifold paths taken together, generates a higher dimensional surface that approaches or departs the reference orbit. Moreover, these manifolds appear as two-dimensional surfaces when projected onto three-dimensional configuration space (position only). An example of a section of one of these surfaces in an L_1 centered rotating frame appears in Figure 1 for stable manifolds associated with an L_1 Lissajous trajectory.[15] Initially, the manifold surface contracts as the manifolds approach the Earth from the bottom of the figure. (The surface is "twisting" as it wraps around the region near the Earth on the right edge of the figure.) After passing the Earth, it expands before contracting again as it approaches the libration point orbit. As time increases, the manifold surface approaches the LPO and, in fact, is virtually indistinguishable from the Lissajous trajectory; this is consistent with the asymptotic nature of the manifold structures. States that reach any point on this surface will asymptotically approach the Lissajous orbit as they evolve, provided that each of the elements of the state equals the corresponding element of the seven-dimensional state (position, velocity, and time) on the manifold at the specified point.

To develop a usable approximation for an Earth-to-halo transfer, a single trajectory is selected along the surface representing the stable manifold. The trajectory must pass appropriately close to the Earth, and may include a lunar encounter, if desired. This solution serves as the initial approximation to the transfer from the vicinity of the Earth to the libration point orbit. In general, however, the Earth close approach will not satisfy the necessary transfer trajectory injection (TTI) constraints, such as altitude and/or inclination, nor will it include phasing orbits. Thus, the methodology described in Howell et al.,[10]

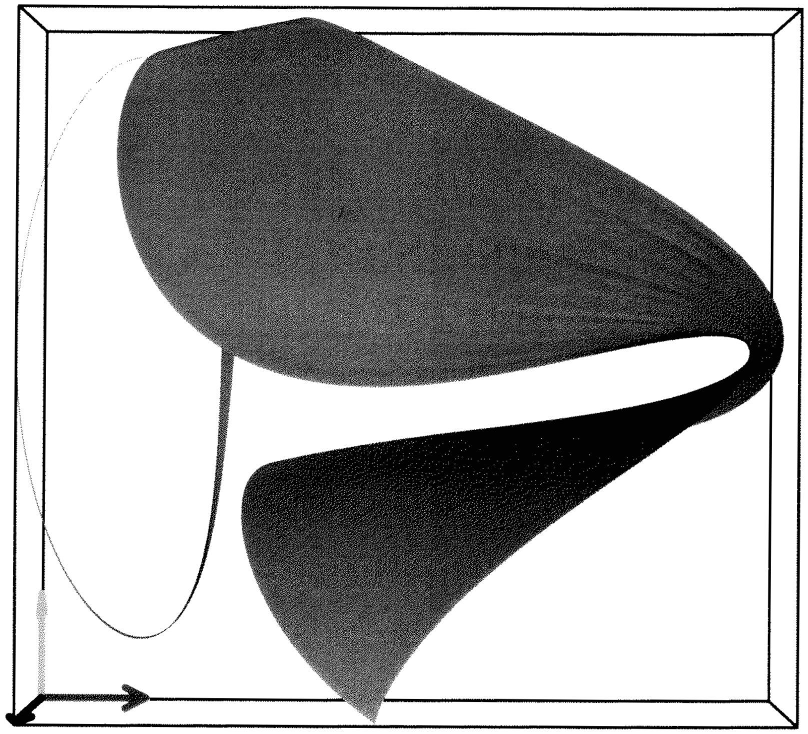

Figure 1. Portion of the Stable Manifold Projected onto Configuration Space (Courtesy: Brian Barden, Purdue University)[15]

Wilson,[14] and Howell and Wilson,[16,17] is employed to enforce the constraint conditions that will exist at the pre-determined transfer injection point; it also allows the inclusion of any desired number of phasing loops to set up a lunar encounter. Of course, the methodology must then determine the maneuver required to match the state defined on the manifold.

Two-Level Corrections Process

This general procedure is essentially a two level differential corrections scheme. First, a final target state from an appropriate single trajectory on the manifold surface is selected. (Recall that the stable manifold of choice is the one that passes closest to Earth, or is otherwise determined to provide a reasonable target for the transfer path.) The transfer trajectory is then defined as the path from the transfer injection point to the final target point on the numerically determined path that approximates the manifold. This initial approximation to the transfer trajectory is discretized into a series of target states (also called patch points) along the path. Between consecutive patch points, a simple differential corrector is used to ensure position continuity at the final point, by varying the velocity at the previous state. Application of this first level corrector results in a complete transfer that is continuous in position and time, but may have velocity discontinuities at *each* patch point. Based on these velocity discontinuities, as well as any constraint violations, a set of position and time corrections are determined for each target state along the trajectory. These state changes are applied concurrently to the set of target states describing the transfer path, and the level one corrector is again used to enforce position continuity throughout the new set of target states; this constitutes the second level of the differential corrections procedure. This two-step iterative process ultimately leads to a trajectory that is fully continuous in position and velocity (with the possible inclusion of deterministic maneuvers) and satisfies all constraints placed on the solution (such as initial inclination, altitude, or final state targets). Further details on this methodology are available in various references.[10,14,16–18]

This methodology is now specifically applied to the transfer problem in which the specific single trajectory that asymptotically approaches the libration point orbit is pre-determined. Thus, a series of target states are selected along the trajectory that numerically represents the desired stable manifold, and a final state is identified to serve as a first guess for the target point at the end of the transfer. The fixed position and time corresponding to this final state on the manifold surface are targeted by the two level differential corrections process to produce the complete transfer trajectory. In order to precisely approach the desired libration point orbit, the final state on the transfer must, in fact, lie on the surface that is representative of the stable manifold. The position and time requirements associated with the final state are met by this differential corrections procedure, however, the velocity state is not constrained in the solution process. Therefore, a maneuver is required to correct any velocity discontinuity between the velocity along the transfer at arrival and the required velocity state on the manifold. This maneuver arises because only position and time are targeted during the corrections process, and is denoted the Libration Orbit Insertion or LOI. Once the state of the vehicle is actually on the representative surface, it approaches the libration point orbit, asymptotically. This completes the transfer from Earth to the LPO with, theoretically, no additional maneuvers.

METHODOLOGY FOR OPTIMAL LOI SELECTION

Utilizing the procedure detailed above, the final target state is pre-determined to be on the surface representing the manifold, and, thus, results in a transfer that approaches the LPO, but may or may not correspond to a solution with an acceptable LOI cost. A methodology is sought to allow this "fixed" LOI target state to vary along the two-dimensional surface to minimize the required insertion maneuver, so that the resulting transfer trajectory still inserts onto the same surface and, hence, approaches the desired libration point orbit asymptotically. Schematically, this is depicted in Figure 2. Initially, the final target state $\bar{X}_{act} = (\bar{R}_{act}, t_{act})$ for the transfer lies on the desired manifold surface in position and time, but requires some associated LOI cost (i.e., arrival $\Delta\bar{V}_N$) to achieve the seven-dimensional manifold state that will approach the libration point orbit. (Note that subscript N denotes the final target state along the transfer path.) Based on this velocity difference ($\Delta\bar{V}_N$), a change in the position and time elements of the arrival state, $\Delta\bar{X} = (\Delta\bar{R}_N, \Delta t_N)$, is calculated to reduce the magnitude of the required maneuver. Ideally, since all points on the manifold asymptotically approach the libration point orbit, this process simply seeks to shift the final target to *any* point on the representative manifold that results in a lower LOI cost. Computation of this shift in the state is based on the following sensitivity partials,[14]

$$\frac{\partial \Delta\bar{V}_N}{\partial \bar{R}_N} = -B^{-1}_{N-1,N}\, A_{N-1,N} \;, \tag{1}$$

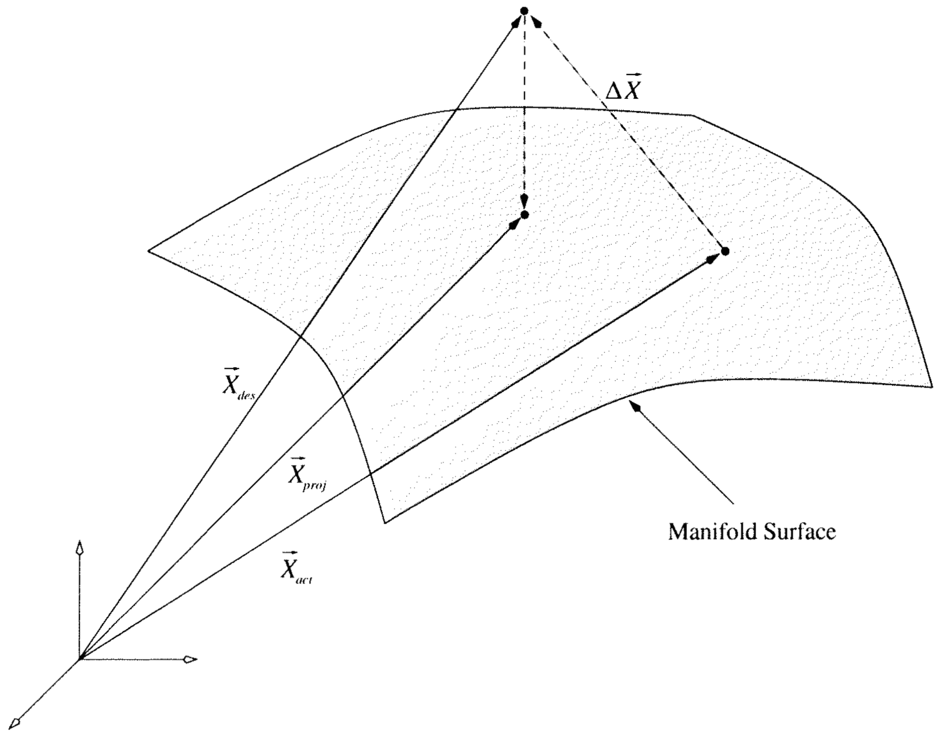

Figure 2. Stylized Representation of Manifold Targeting Procedure

$$\frac{\partial \Delta \bar{V}_N}{\partial t_N} = \bar{a}_N + B_{N-1,N}^{-1} A_{N-1,N} \bar{V}_N \quad , \tag{2}$$

$$\frac{\partial \Delta \bar{V}_N}{\partial \bar{R}_{N-1}} = B_{N-1,N}^{-1} \, , \tag{3}$$

$$\frac{\partial \Delta \bar{V}_N}{\partial t_{N-1}} = -B_{N-1,N}^{-1} \bar{V}_{N-1} \quad , \tag{4}$$

where the subscript $N-1$ corresponds to the target point immediately prior to the final state, and $\bar{V}_N$ and $\bar{a}_N$ represent the velocity and acceleration vectors corresponding to the final target state N. The matrices $A_{N-1,N}$ and $B_{N-1,N}$ are 3x3 submatrices of the state transition matrix relating changes in the state at $N-1$ to changes in state N. A more complete discussion of these partials and their application in the two level differential corrections process can be found in Wilson,[14] and Howell and Wilson.[16,17]

Applying the position and time corrections, $\Delta\bar{X}$, to the original target state $\bar{X}_{act}$ results in a new final state $\bar{X}_{des}$ that, in all likelihood, does not lie on the required surface. This condition results, of course, because manifolds represent solutions to the nonlinear equations, but the corrections scheme is an inherently linear process. To compensate, this new final state is projected back onto the manifold surface; another state $\bar{X}_{proj}$ is then obtained that *does* lie on the desired surface, and therefore, is an acceptable final target state. A new transfer is determined to this new LOI location that should require a smaller maneuver to insert onto the manifold. This iterative process is repeated until some minimum cost is achieved.

ONE DIMENSIONAL VARIATIONS ALONG THE MANIFOLD

As an application of this methodology, consider the variation of the LOI target state along a single manifold trajectory. In this case, the "surface" is, in fact, one-dimensional, corresponding to a single one-dimensional stable manifold that is identified as an acceptable path to the LPO. Some LOI target state $\bar{X}_{act}$ along the one-dimensional trajectory is selected and the transfer is computed to meet the desired position and time at the fixed transfer end state. Given the cost associated with this computed solution, corrections in the final state ($\Delta\bar{X}$) are determined using Equations (1) – (4) that reduce the velocity error at arrival. This update to the state is added to the previous final target point to produce a new final target state $\bar{X}_{des}$; however, this point no longer lies on the desired manifold trajectory.

By projecting $\bar{X}_{des}$ onto the representative one-dimensional stable manifold, a new final target state $\bar{X}_{proj}$ is determined. This projection is computed by minimizing the distance from the desired target, $\bar{X}_{des}$, to some point along the actual trajectory. Once the new LOI target state is determined from $\bar{X}_{proj}$, a new transfer is computed using the previous solution as an initial guess. A new LOI cost is computed and the process is repeated until some minimum insertion cost is achieved. For projection of the desired end state onto the surface, time is selected as the independent variable along the manifold. The time along the one-dimensional path is monotonic and provides a one-to-one mapping along the trajectory, i.e., there is only one state associated with each time along the one-dimensional manifold. To ensure an adequate resolution for the time variable along the path, a 10^{th} order interpolation scheme is used with nodes selected once every day along the numerically integrated path. This proves to be an efficient method for both storage and evaluation of the representative manifold states over a given time interval.

Transfer to an L_2 Libration Point Orbit

As an example of this process, consider the direct transfer from Earth to a Lissajous orbit in the vicinity of the Sun-Earth L_2 point using a single lunar gravity assist. The one-dimensional manifold selected for this analysis is plotted in Figure 3; the coordinate frame is centered at the Earth and rotates with the Earth about the Sun, such that the x-axis is always directed along the line from the Sun to the Earth. In the figure, the path representing the manifold extends from the lunar orbit (just beyond the lunar encounter) to the state on Julian date 2454560.0, approximately half way through the first revolution along the Lissajous trajectory. The square symbols on the plot denote 10 day intervals originating at JD 2454380.0, just after the lunar encounter. The "nominal" LOI point at JD 2454400.0 is also marked. This is the initial guess for the fixed state, selected as a baseline to initiate the analysis. Experience suggests this region along a one-dimensional stable manifold to be a reasonable approximation for the location of an insertion point that satisfies the launch constraints for a competitively low cost. Note that, previous analyses in this problem have actually used the x-axis crossing points for insertion (roughly corresponding to JD 2454490 in the figure). This choice was based as much on the design process and available tools as on the dynamical analysis.[2,4,5,7] From the methodology applied here, the solution resulting from application of the differential corrections process is presented in Figure 4. In this case,

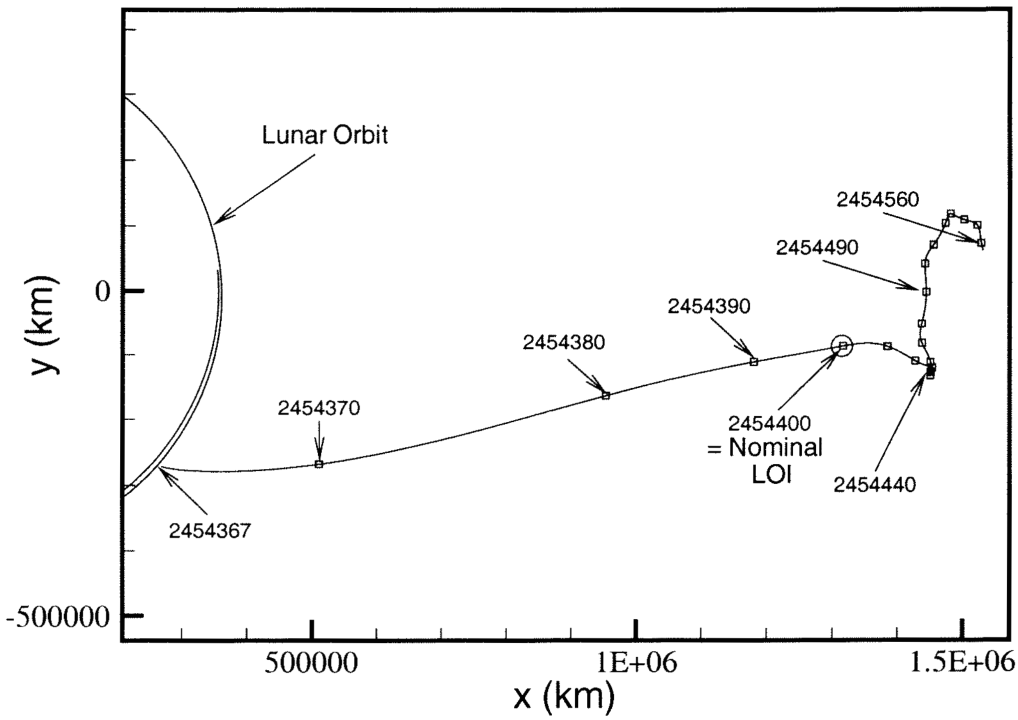

Figure 3. Selected Manifold Trajectory for Earth-to-L_2 Transfer Example

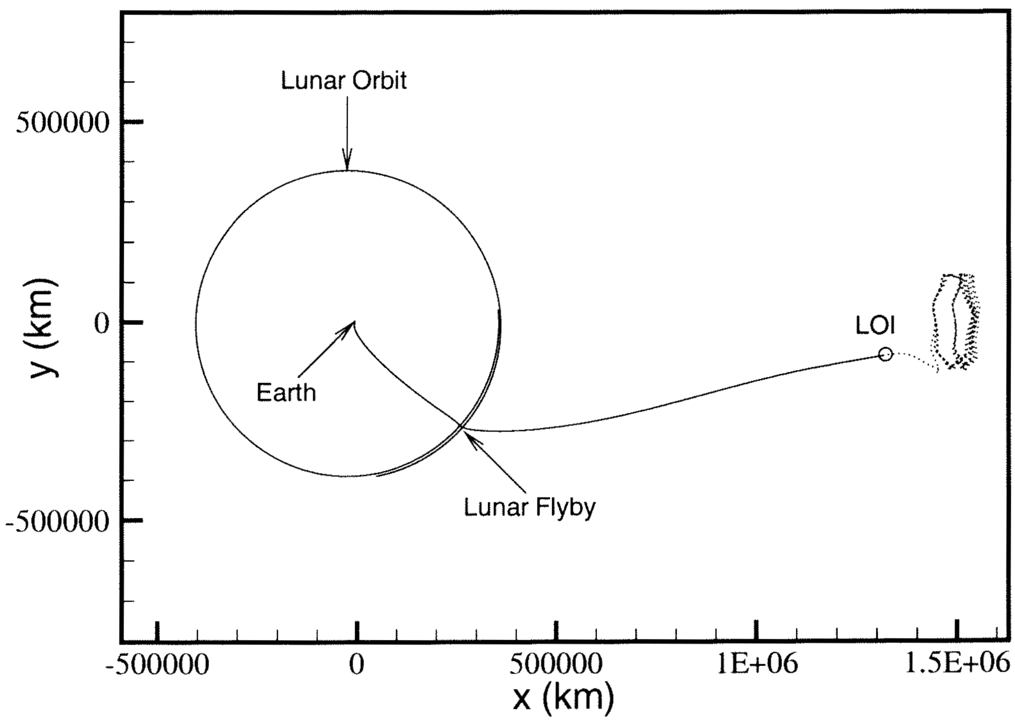

Figure 4. Earth-to-L_2 Transfer Using Zero Phasing Loops

an initial perigee of 200 km altitude and an inclination of 28.5 deg results in a transfer with an LOI cost of 1.93 m/s.

To isolate the effects of the variations in the LOI target point location on the LOI cost, the transfer trajectory injection date is fixed at some value that is within the range identified for the given nominal. After the initial trajectory is determined, the LOI target state variation scheme is applied to this solution. The results of this procedure are presented in Figure 5 for the direct transfer case (i.e., a transfer with no phasing loops). In the figure, the LOI maneuver cost is plotted as a function of the LOI date for a series of transfer injection dates. (To clarify the figure, the abscissa corresponds to the LOI target Julian date minus 2454000.) Each curve in the figure then represents the variation in LOI cost for a specified transfer injection date; for example, the curve labeled 62.5 corresponds to solutions with transfer injection on JD 24543(62.5). The minimum LOI cost determined by the LOI target state variation procedure for each given TTI date curve is marked with a diamond. These minimums are connected by a dotted line to signify that a continuum of solutions is possible over this limited range of transfer injection dates. Note that the overall minimum LOI cost that results from this procedure over the given range of injection and LOI target dates is 0.31 m/s on JD 2454(412.058), corresponding to a transfer injection date of JD 24543(63.764). As the TTI date varies, the LOI cost for a given target state along the representative manifold decreases to a minimum and then increases again, as seen along each curve in Figure 5. Note also that for TTI dates near JD 2454363.764 (corresponding to the solution curve with the minimum cost) the LOI costs are fairly constant over a 30 to 40 day range from JD 2454380.0 to JD 2454420.0. As the TTI date varies from this minimum value, the LOI cost rises rapidly and the variations along a given curve lose their linear nature.

For LOI target dates beyond JD 2454420.0, the LOI cost begins to rise dramatically. To demonstrate this fact, the lowest curve in Figure 5 corresponding to the minimum LOI cost (associated with TTI on JD 2454363.764) appears in Figure 6 for an expanded range of LOI target dates from JD 2454370.0 to 2454560.0. (Refer to Figure 3 for the correlation between the LOI target dates and positions along the manifold.) Notice that, aside from the two "spikes", the LOI cost is fairly constant over the entire range along the one-dimensional manifold. This indicates that the precise location of the LOI maneuver is not as critical as the selection of the TTI date in order to achieve a desirable insertion cost, at least for the direct transfer case.

It is speculated that the large increases in LOI cost around JD 2454440 and 2454525 may be a function of the differential correction process used to obtain the solutions. Near these locations, it becomes increasingly difficult to determine a satisfactory transfer. The reason for the difficulty is unclear, but may be a function of the geometry of the manifold surface (in relation to the ecliptic plane, for instance). Between the spikes, a second local minimum exists near JD 2454480.0. This LOI cost is actually 0.09 m/s lower than the previous minimum. It is likely that any science activities for the mission would be underway at this point; thus, to avoid any maneuvers in this region, the earlier local minimum is selected for analysis.

LOI Target State Variations with Phasing Loops

Similar to the direct case, the LOI target state variation procedure is also applicable

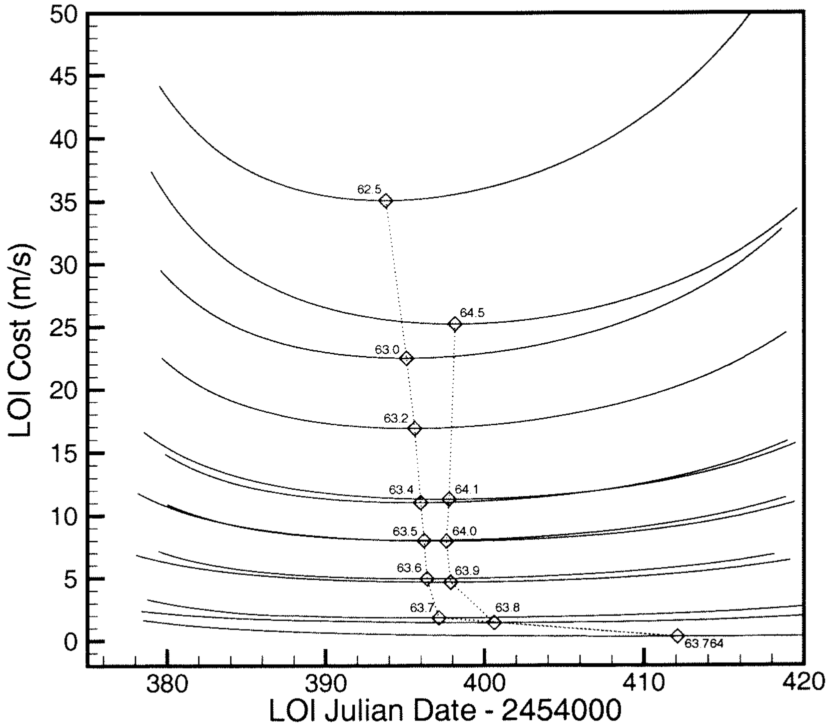

Figure 5. LOI Target Date Variation for Zero Loop Case

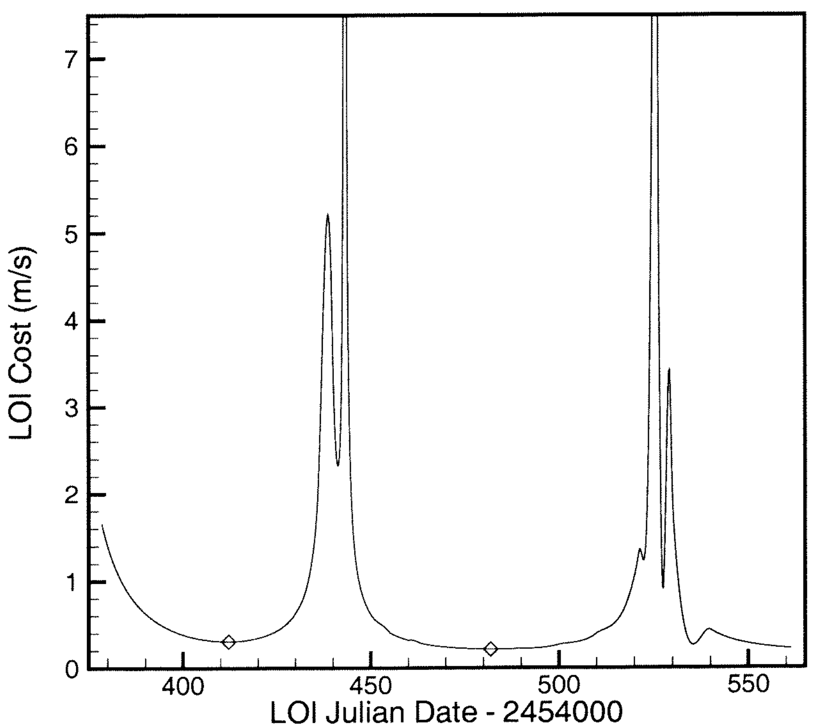

Figure 6. Minimum LOI Target Date Variation for Zero Loop Case

to transfers that include multiple phasing loops. The results from application of the one-dimensional manifold targeting procedure are presented in Figure 7 for a transfer with two phasing orbits prior to the lunar encounter. Again, each curve corresponds to a specific transfer injection date that is referenced to JD 2454300. Solutions along a single curve, then, represent the relationship between LOI cost and the position of the LOI target state (identified by the associated date). Note again that the lowest curve, corresponding to the lowest LOI costs is fairly flat. As the TTI date increases, the minimum LOI cost along each curve (denoted with diamond symbols) reaches a minimum if TTI occurs on JD 24543(37.3), followed by a local maximum corresponding to TTI on JD 24543(39.5). Although it appears that there may be two separate types of solutions, there is, in fact, only one transfer type represented here, with continuous solutions throughout the valid launch period. The minimum cost of 2.23 m/s corresponds to a solution with an LOI that occurs on JD 2454401.308, and has a corresponding transfer injection date on JD 2454337.3. As seen previously, the lowest curve, the one that includes the solution with the minimum cost, is very flat in this region. However, as before, the cost begins to increase as the LOI dates approach JD 2454440, the region of the first "spike" seen in Figure 6.

A comparison between the minimum cost solution, as indicated in Figure 7, and the solution corresponding to the original "nominal" LOI location on JD 2454400.058 appears in Figure 8. The original LOI cost associated with the path that includes two phasing loops is 30.35 m/s (denoted by the dashed line), while the trajectory represented by the solid line in Figure 8 incorporates a "best" LOI maneuver magnitude of 2.23 m/s. So, application of this targeting algorithm reduces the cost of the new transfer by 28 m/s over the original solution. Similar results have been achieved for transfers that include one and three phasing loops as well.

Application to the GENESIS Trajectory Design

If all goes according to schedule, in early 2001 the GENESIS spacecraft will be launched with the goal of returning samples of the solar wind to the Earth for detailed investigation into the origins of the solar system. The GENESIS trajectory includes a Lissajous orbit in the vicinity of the Sun-Earth L_1 point (between the Sun and Earth) to facilitate the collection of particles from the Sun over a two and a half year mission. Dynamical systems theory is used in the design of the GENESIS trajectory[6,10] (Figure 9) to determine both the transfer to the Lissajous, as well as the return leg that extends toward the Sun-Earth L_2 point leading to a daytime reentry over Utah. By exploiting the natural dynamics of the Sun-Earth-Moon system, the entire trajectory can be executed with just a single deterministic maneuver at LOI, roughly three months after launch.

For application to this mission, the one-dimensional manifold targeting process is extended to determine a more optimal location for the LOI maneuver. Unlike the previous L_2 transfer case, the GENESIS trajectory incorporates a direct transfer out to its L_1 Lissajous, with no lunar encounter. This actually permits a broader region of the representative manifold surface to be investigated in the search for possible LOI locations, since the transfer is not constrained by the timing of the lunar flyby. A series of one-dimensional trajectory arcs associated with the appropriate revolutions along the desired Lissajous orbit, then, can be numerically computed that, taken together, approximates a two-dimensional surface that reflects a stable manifold approaching the periodic reference orbit. A subsection of this

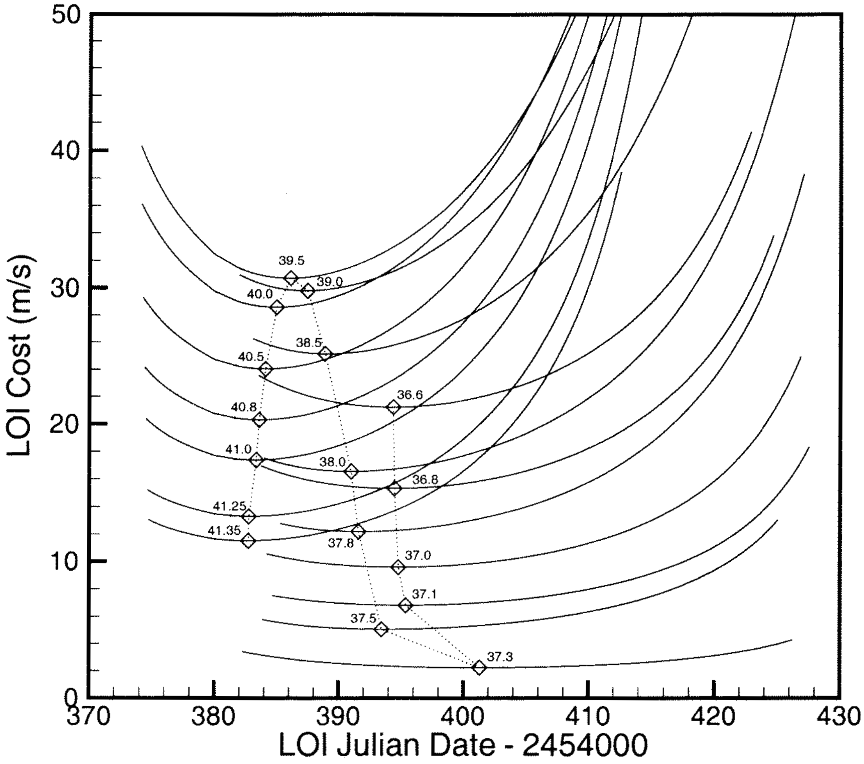

Figure 7. LOI Target Date Variation for Two Loop Case

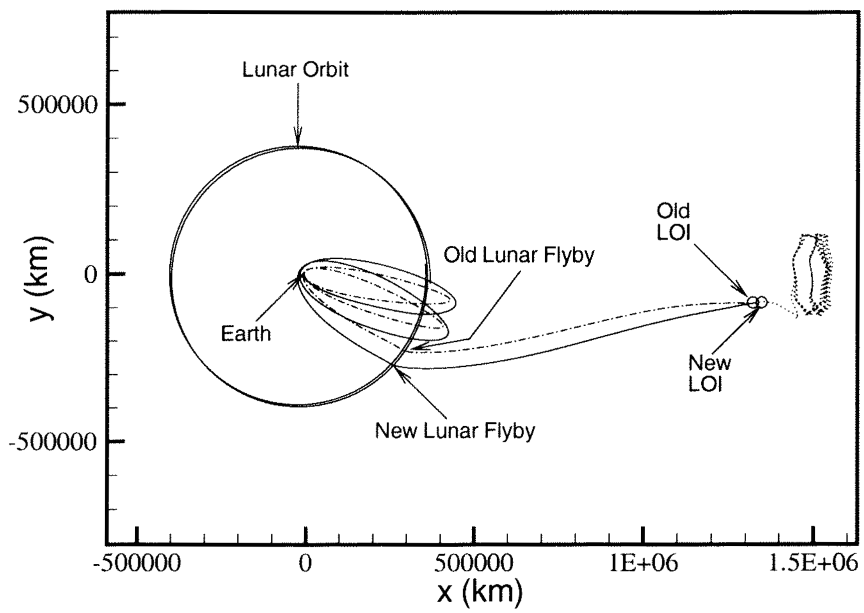

Figure 8. Nominal and Best Solutions for Two Loop Case

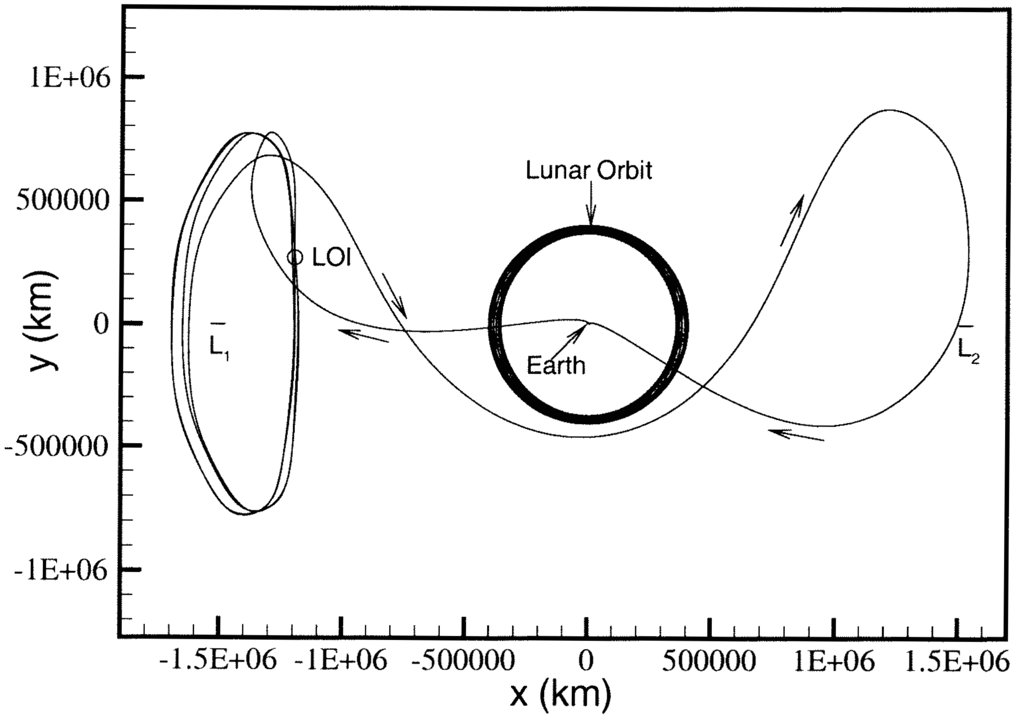

Figure 9. GENESIS Trajectory for January 2001 Launch

surface appears in Figure 10. The surface has been constructed from sixty trajectories, and is bounded by the trajectories labeled #140 along the lower edge of the surface to #200 along the upper edge. Notice that, as in Figure 1, the surface contracts as it approaches the Lissajous orbit and becomes indistinguishable from the reference orbit after roughly one revolution (about 180 days). As the surface evolves in time, a smaller subsection of the surface, labeled Region A, has been highlighted in the figure. The red lines in this region are contours of constant epoch, beginning on JD 2451940 (January 30, 2001) and extending through JD 2451975 (March 6, 2001). By utilizing the manifold targeting procedure individually on each of the sixty one-dimensional trajectories over the given time frame, an estimate of the LOI cost necessary to insert onto this portion of the manifold surface is obtained.

As in the examples of transfers to L_2, the transfer trajectory insertion (TTI) date must be fixed to isolate the effects of changing the date corresponding to the LOI target state on the LOI maneuver cost. Therefore, a TTI date on January 7, 2001 (the opening of the launch period) is selected for this analysis, along with constraints on perigee and inclination, i.e., a 185 km altitude perigee and a 28.5 deg orbital inclination for launch from the Eastern Test Range. The results of the targeting scheme in Region A for the January 7 TTI date appear in Figure 11. In this figure, the x-axis is the manifold ID number, while the y-axis represents epochs along the given trajectory. The color contours in the figure, then, correspond to various levels of LOI cost ranging from 23 m/s (dark blue) to greater than 134 m/s (red). There is a small range of LOI locations roughly through the middle of the selected region that represent acceptable LOI costs, that is, under 40 m/s. By examining the contour plot, it is immediately obvious that LOI targets states on trajectory #168 from

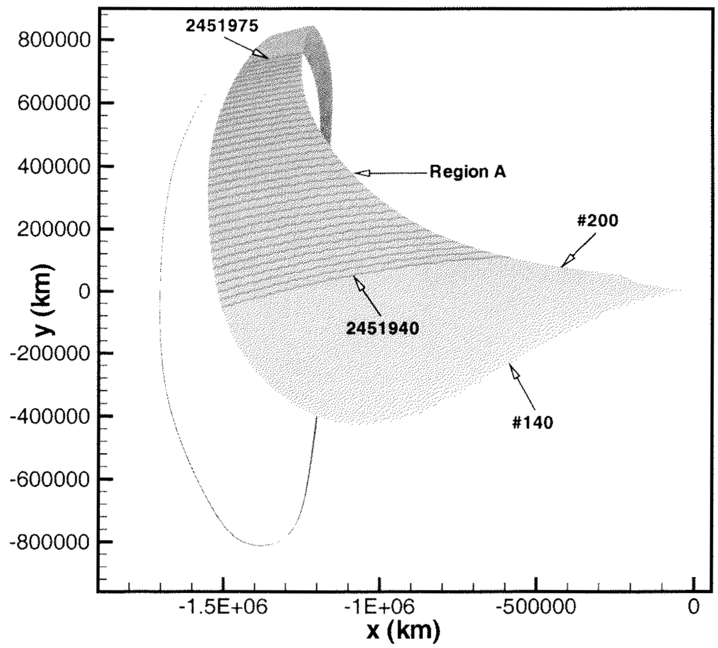

Figure 10. Definition of GENESIS Manifold Region A

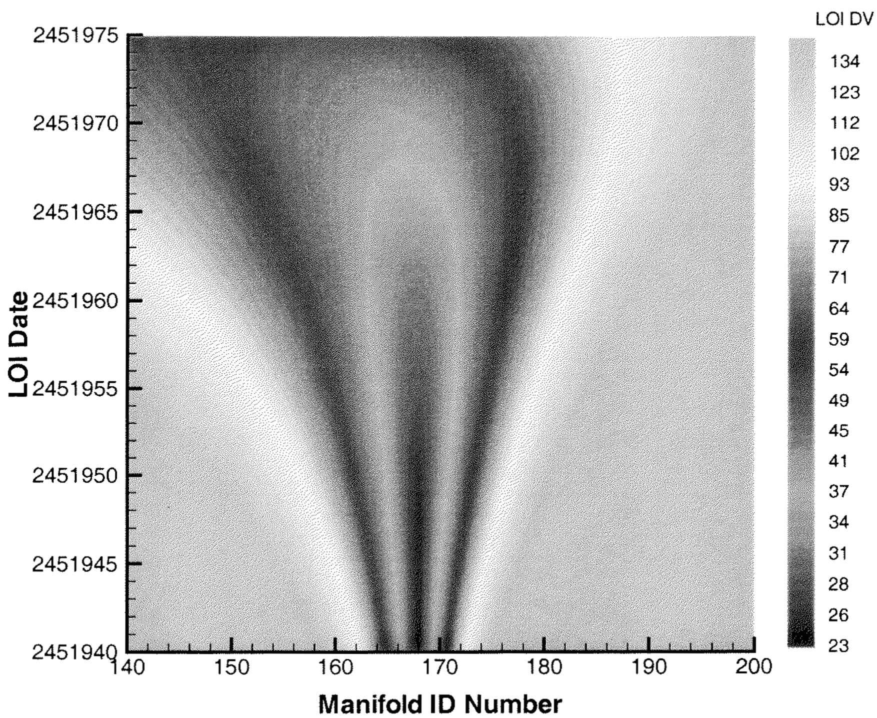

Figure 11. Insertion Cost for GENESIS Manifold Region A

JD 2451940 through JD 2451960 are excellent candidates for possible insertion locations.

For dates beyond JD 2451975, the cost rises again, similar to the trends seen in Figure 6. Therefore, this segment of the surface is bypassed in favor of a second region corresponding to dates JD 2452000 (March 31, 2001) through JD 2452060 (May 30, 2001). This section of the surface is labeled Region B and is depicted in Figure 12. (For clarity, earlier portions of the surface have been removed from the figure to highlight the desired region.) Notice that by this point in time, the surface has contracted substantially and the individual trajectories are much less distinct. It is not surprising, then, that application of the targeting process results in broad bands of opportunity that are similar in LOI cost, as seen in Figure 13. In this case, a large region of acceptable LOI costs (< 40 m/s) exists between JD 2452000 and JD 2452045 over most of the trajectories. These kinds of analyses ultimately led to the selection of an LOI location on 2452030 (April 30, 2001) for the nominal GENESIS trajectory corresponding to a launch in January 2001. Although the final LOI cost for the nominal mission is, in fact, slightly larger that the minimum 23 m/s due to other design factors, the qualitative information from this kind of surface targeting is particularly useful in making better design decisions in regard to LOI placement.

CONCLUSIONS

This manifold targeting procedure is highly applicable to a variety of libration point missions, such as the ones depicted here. This process has proven useful in the GENESIS mission design to determine a more optimal location for the Lissajous orbit insertion maneuver. Extension of the one-dimensional results to the full two-dimensional surface should allow an optimal LOI location to be determined through a search over the entire manifold surface in one step, while maintaining the desired characteristics of the libration point orbit.

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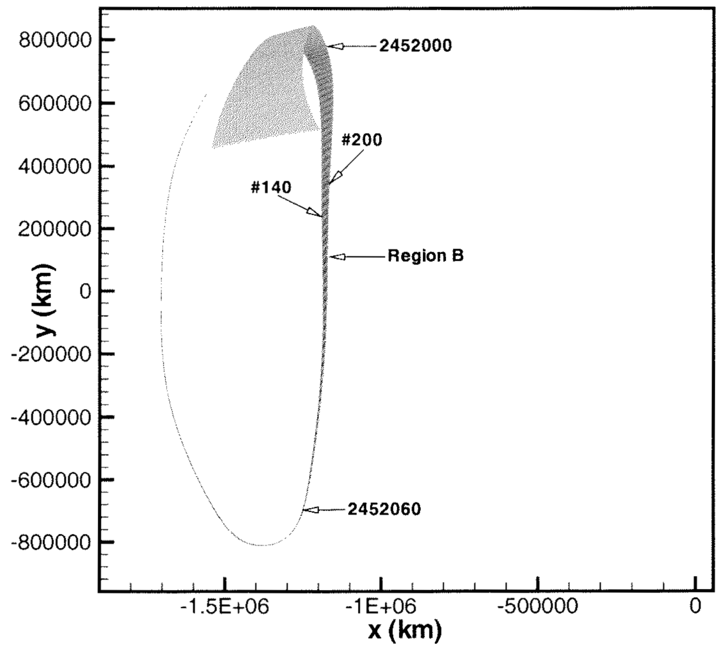

Figure 12. Definition of GENESIS Manifold Region B

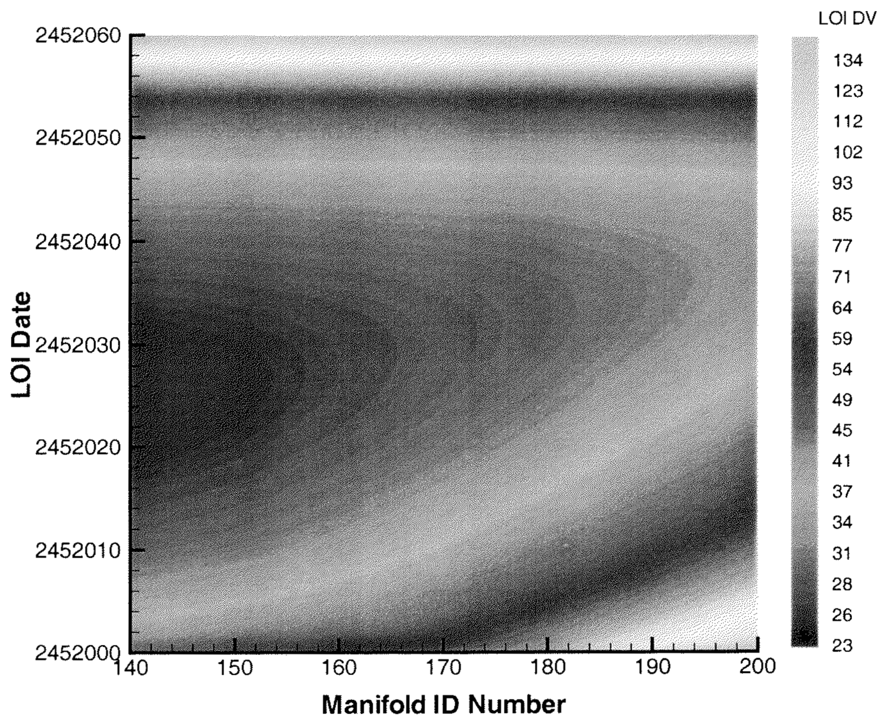

Figure 13. Insertion Cost for GENESIS Manifold Region B

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